



STUDY ON GRAPH THEORY CONCEPTS AND ITS APPLICATIONS

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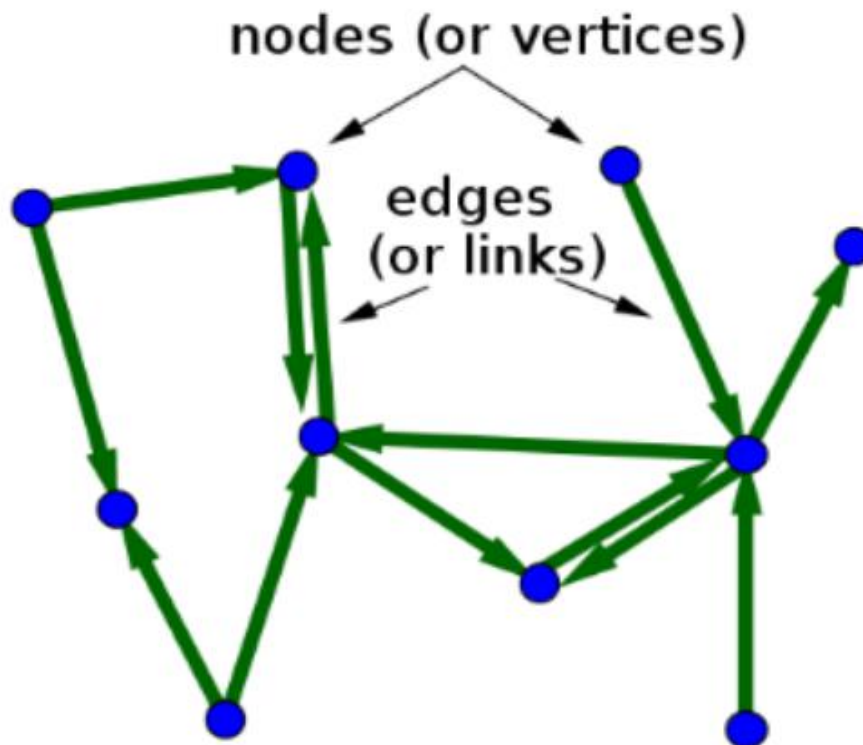
ABSTRACT

The primary goal of this work is to present the fundamental principles of graph theory and to investigate linked graphs, Eulerian graphs, and Hamiltonian graphs, among other topics. Graph theory is a discipline of mathematics that has numerous applications in both mathematics and other fields of science. The purpose of this study is to highlight the applications of graph theory in our daily lives, such as in mathematics, computer science, and operations research.

Keywords: *Graph, Application, Mathematics, Research, Linked graphs*

INTRODUCTION

Euler created graph theory in 1736 in order to solve the Konigsberg bridge problem. Graph theory is a branch of mathematics that deals with the study of graphs. In the case of information including relationships between things, a graph provides a useful means of describing that information. The items are represented by vertices, while the relationships between them are represented by edges. In general, a graph is represented as a collection of vertices (also known as nodes or points) that are linked together by edges (arcs or line). The graph containing the vertices V and edges E is denoted by the letter G . (V,E) . Each edge in an undirected graph G is associated with an unordered pair of vertices, and each vertex in an undirected graph G is associated with an unordered pair of vertices. A directed graph is a graph that contains a collection of objects (referred to as vertices or nodes) that are connected to one another by edges that are all directed from one vertex to another. A directed graph is sometimes referred to as a digraph or a directed network in some circles. An undirected graph, on the other hand, is a graph in which the edges are not bidirectional in any way. When sketching a directed graph, the edges are often represented by arrows showing the direction of the graph, as shown in the following picture.

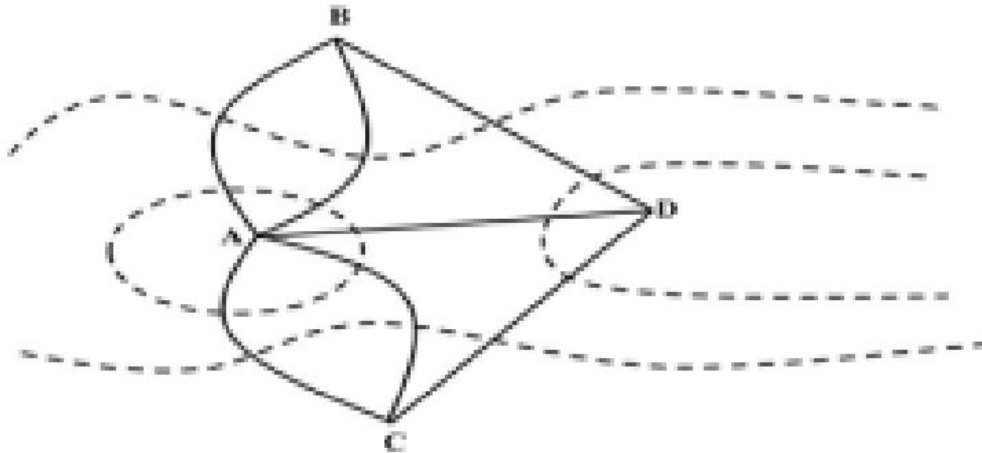


A directed graph with 10 vertices (or nodes) and 13 edges.

As an added bonus, a simple graph is defined as a graph that does not contain any self loops and no parallel edges, whereas a pseudograph is defined as a graph that contains self loops and parallel edges. Aside from that, graph theory is employed in a wide variety of fields such as communications, engineering, physical sciences, among others. Graph theory is also extremely valuable in areas of computer science such as switching theory and logical design, artificial intelligence, and computer graphics, to name a few.

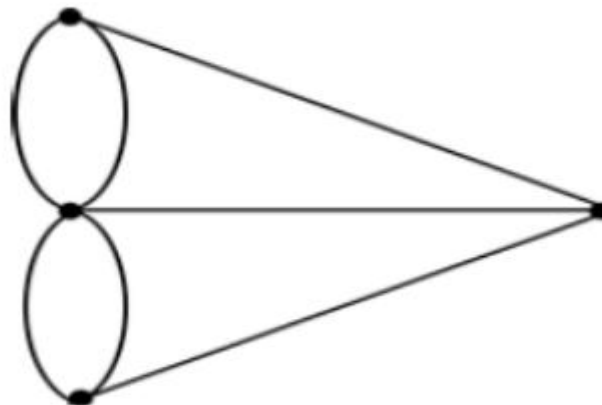
THE ORIGIN OF EULERIAN GRAPH

The beginning of graph theory may be traced back to 1736, when Leonhard Euler solved an issue that had been perplexing the good residents of the town of Königsberg in the German state of Prussia (now Kaliningrad in Russia). The river Pregel flows through the Russian city of Königsberg, separating the city into four geographical divisions, two of which are river banks and two of which are islands or delta formations. The river Pregel is named after the city's founder, Prince Königsberg. Seven bridges united the four land regions of the world.



The bridges of Königsberg problem

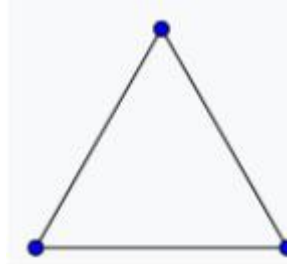
Specifically, the participants wanted to discover if it was possible to start at any position in town, cross every bridge exactly once, and then return to the starting point. By employing the vocabulary of points (which represent the land regions) and lines, Euler suggested and further emphasised that it is impossible to accomplish this task (representing the bridges). He abstracted the example of Königsberg by removing all of the features that were superfluous. He made an image consisting of "dots" that represented the landmasses and line-segments (edges) that represented the bridges that connected those landmasses, and he labelled the artwork with the word "bridges." The resulting image was fairly similar to the one illustrated in the illustration below.



For each given graph, Euler claimed that it is possible to depict it with each edge visited exactly once only if and only if it has either zero or exactly two nodes with odd degrees. The graph that satisfies this requirement is referred to as an Eulerian circuit or path. That is to say, a simple path in a graph G is referred to as an Euler path if it crosses each and every edge of the graph only once.

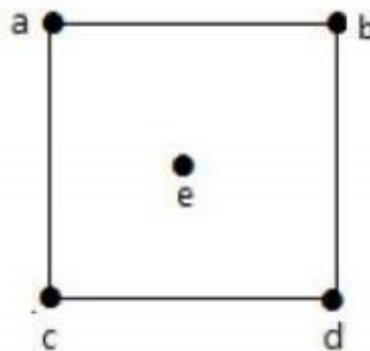
DEFINITIONS OF ESSENTIAL TERMS

1) Complete the graph; and 2) A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a single edge, and every pair of distinct edges is connected by a single edge. It is also referred to as the universal graph. As an illustration,



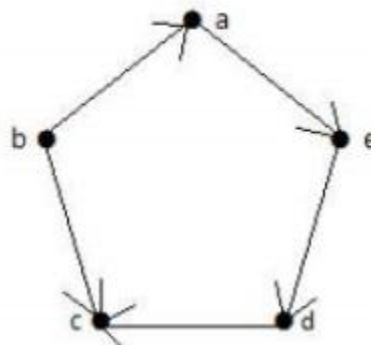
a directed graph in which every pair of different vertices is connected by a pair of distinct edges is known as a complete digraph (one in each direction). If there are more than one vertex close to one vertex V , the graph is said to have a higher degree. It is symbolised with the letter $\text{deg}(V)$. In a simple graph with n vertices, the degree of any vertex is $\text{deg}(V) \leq n - 1 \forall V \in G$.

A vertex can create an edge with all other vertices except for the one with which it is associated. As a result, the degree of a vertex will be equal to the sum of the number of vertices in the graph less one. This 1 is for the self-vertex, which cannot construct a loop on its own since it lacks the necessary geometry. In the case where any of the vertices is connected by a loop, the graph is not considered to be Simple. In an undirected graph, the degree of a vertex is defined as the number of directed edges the vertex has. Example No. 1 Please take a look at the following chart:



In the graph above, $\text{deg}(a)$ equals 2, $\text{deg}(b)$ equals 2, $\text{deg}(c)$ equals 2, $\text{deg}(d)$ equals 2, and $\text{deg}(e)$ equals 0. The vertex 'e' is a vertex that has been isolated. Neither a pendent nor a non-pendent vertex can be found in the graph. A pendent vertex is a vertex that has one degree of freedom. Isolated vertex is a term

used to describe a vertex with degree zero. The degree to which a vertex in a directed graph is oriented- Each vertex in a directed graph has two degrees of freedom: an indegree and an outdegree. The indegree of a graph is the number of edges that are entering into a vertex V . The indegree of a vertex V is the number of edges that are coming into the vertex V . Outdegree of a Graph-Outdegree of a vertex V is the number of edges that originate at the vertex V and terminate at another vertex V . Exemple No. 2 Please take a look at the following directed graph for more information. Edge 'ae' extends outwards from the vertex 'a' and is connected to it. As a result, its outdegree is one. In a similar vein, the graph contains an edge 'ba' that leads to vertex 'a'. As a result, the indegree of 'a' is one.



APPLICATION OF GRAPH THEORY

In these days, Google Maps is a very important tool for travelling anywhere in the world, and there are numerous applications for it. We can identify all possible routes from one location to another using Google Maps, as well as the shortest path between two locations. If we regard Google Maps as a graph, the places may be thought of as vertices and the routes as edges, respectively. So, when we use the Google Maps software to identify the shortest route between two points, it will find all edges or vertices that connect these two points or vertices and will also present the shortest edge as a shortcut to the destination.

Internet applications: The internet is a very beneficial innovation of modern science that has a wide range of applications. The notions of graph theory are applied in the operation of the internet's functioning technique. When it comes to internet connectivity, all of the users are referred to as vertices, and the connections that connect them are referred to as edges. An analogous situation exists in the case of social networking sites, where one buddy is connected to other people and his friends are likewise connected to others. A graph will be formed if we take the friends to be the vertices of a graph and define an edge between them.

Chemical Applications of Graph Theory: Graph theory is employed in the field of chemistry to aid in the mathematical modelling of chemical events. We may create a natural model of a molecule, in which the vertices represent the atoms and the edges represent the bonds between them. Chemical graph theory (CGT) is a branch of mathematical chemistry that works with the nontrivial uses of graph theory to address molecular problems. It is a subfield of mathematical chemistry. In operation research, graph theory is a very helpful tool. Graph theory is used in a variety of applications. Graphs can be used to answer some operations research challenges, such as those involving graphing. When it comes to transportation problems, the graph theoretical approach is quite effective when it comes to minimizing transportation costs or maximizing profits.

Moreover, it is employed in a variety of assignment challenges, such as allocating various employees to different occupations, managing the timetable for school or college, and so on. Applications in Computer Science: Graph theory plays an important role in the field of computer science. The algorithms of various programmes are developed with the help of graph theory principles. There are a few algorithms, for example.

- (1) The algorithm for finding the shortest path through a network.
- (2). Identifying the shortest spanning tree.
- (3). Algorithms for locating cycles in a graph, and so on.

CONCLUSION

The primary goal of this study is to demonstrate the significance of graph theory in several disciplines of science as well as in our daily lives as a whole. This paper is useful for students and researchers who want to gain an overview of graph theory and its applications in a variety of domains such as everyday life, mathematics, computer science, operations research, and chemistry, among other things.

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